On neutrino mass in left-right symmetric theories

Michele Frigerio
Service de Physique Théorique - CEA/Saclay

with Evgeny Kh. Akhmedov,
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A master equation

\[ m_\nu = v_L f - v^2 y (v_R f)^{-1} y \]

Neutrino mass: oscillations, neutrinoless 2\(\beta\) decay, large scale structures, ...

Electroweak symmetry breaking scale: LHC physics

Neutrino Yukawa coupling: the way Dirac fermions get mass

Sub-eV scale \(v_L\) versus Grand Unification scale \(v_R\): seesaw mechanism

Majorana-type coupling: lepton number violation in a Left-Right symmetric way
Outline

• A theoretical perspective on present and future experimental results on the neutrino mass

• From tiny neutrino masses to energy scales beyond the Standard Model: the seesaw mechanism

• A non-minimal well-motivated framework: models with left-right gauge symmetry

• A bottom-up reconstruction of the super-heavy seesaw sector and its implications for
  ✴ baryogenesis via leptogenesis
  ✴ Grand Unification theories
Status of oscillations data

3 active light neutrinos (no sterile states): a global fit


 Mixing angle | Data             | $\sin^2\theta_{\text{exp}}$ (at 2$\sigma$) | $\sin^2\theta_{\text{Tri-Bi-Maximal}}$
--- | --- | --- | ---
2-3 | Atm - K2K - Minos | 0.50 (1+0.26, 1-0.24) | 1/2
1-2 | Solar - KamLAND | 0.30 (1+0.20, 1-0.13) | 1/3
1-3 | Chooz | 0.000(+0.025) | 0

Mixing angle

Data

$\sin^2\theta_{\text{exp}}$ (at 2$\sigma$)

$\sin^2\theta_{\text{Tri-Bi-Maximal}}$
\[ \Delta m^2_{12} \equiv m_2^2 - m_1^2 = 7.9 \cdot 10^{-5}\text{eV}(1 \pm 0.09) \]

\[ \Delta m^2_{23} \equiv |m_3^2 - m_2^2| = 2.4 \cdot 10^{-3}\text{eV}(1^{+0.21}_{-0.26}) \]

- Future oscillation experiments may measure \( \text{sign}(m_3^2 - m_2^2) \)
- Absolute mass scale \( m_i \) unknown, but constrained by:
  - tritium \( \beta \) decay: \( m_i < 2.2 \text{ eV} \) [Katrin 3 years: \( m_i < 0.2 \text{ eV} \)]
  - neutrinoless 2\( \beta \) decay: \( m_{ee} < (0.3 \div 1.0) \text{ eV} \) [Cuoricino & Nemo-3: \( m_{ee} < 0.1 \text{ eV} \)]
  - cosmological bounds: \( \Sigma_i m_i < (0.4 \div 0.7) \text{ eV} \) [Planck CMB + lensing: \( \sigma(\Sigma_i m_i) \approx 0.05 \text{ eV} \Rightarrow m_i \) determined!]
- If neutrinos are Majorana, two unknown CP violating phases \( \text{arg}(m_1/m_2) \) (enters \( m_{ee} \)) and \( \text{arg}(m_3/m_2) \) (not accessible)
Neutrino mass matrix

The most sound theoretical interpretation of all neutrino data: add to the Standard Model a **3x3 Majorana mass term**

\[ m_\nu = U \text{diag}(m_1, m_2, m_3) U^T , \quad U = U(\theta_{12}, \theta_{23}, \theta_{13}, \delta) \]

- **Knowns**: \( \theta_{12}, \theta_{23}, m_2^2 - m_1^2 \) and \(|m_3^2 - m_2|^2\); upper bounds on \( \theta_{13} \) and \(|m_i|\).
- **Unknowns**: \( \theta_{13}, \text{sign}(m_3^2 - m_2^2), |m_i| \) and three CP phases, \( \delta, \arg(m_1/m_2), \arg(m_3/m_2) \)

The **structure of** \( m_\nu \) provides a crucial clue on the particle theory beyond the Standard Model

Theoretical priorities = fix the largest uncertainties in the structure of \( m_\nu \):

(I) mass spectrum
(II) Majorana CP phases
(III) mixing angles
(IV) Dirac CP phase
What ν physics beyond SM?

A non-zero Majorana neutrino mass may be introduced as the effect of the unique dimension 5 effective operator:

\[
m_\nu = \sum_i m^{(i)}_\nu
\]

A host of new physics candidates brings a contribution to neutrino mass through this operator:

\[
m_\nu = \sum_i m^{(i)}_\nu
\]

EWSB:

\[
<H> = 174 \text{ GeV}
\]
Seesaw means to interpret the effective operator as the exchange of a certain super-heavy particle.

Seesaw Mechanism in 3 possible versions:

[type I] SM singlet fermions $N_R : m_\nu \sim \nu^2 / M_R$

[type II] $SU(2)_L$ triplet scalars $\Delta : m_\nu \sim \nu^2 / M_\Delta$

[type III] $SU(2)_L$ triplet fermions $\Sigma : m_\nu \sim \nu^2 / M_\Sigma$

Minkowski, Gell-Mann, Ramond, Slansky, Yanagida, Glashow, Mohapatra, Senjanovic, Magg, Wetterich, Lazarides, Shafi, Schecter, Valle, Foot, Lew, He, Joshi, Ma
From a mechanism to a theory

Seesaw explains (i) **smallness of $\nu$ mass**
(ii) **baryogenesis via leptogenesis**

However the heavy scale and the new particles are ad hoc...

**minimal Left-Right gauge symmetry:**

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \quad \rightarrow \quad SU(2)_L \times U(1)_Y$$

extensions: $SU_{422}$, $SO(10)$, ...

(i) **right-handed neutrinos** are incorporated naturally
(ii) **maximal parity violation** can be understood
(iii) **Grand Unification** gives a rationale for the heavy scale
(iv) **supersymmetry** can be easily incorporated & **R-parity** is unbroken [if only (B-L)-even Higgs bosons acquire VEVs]
Left-Right symmetric $\nu$ mass

<table>
<thead>
<tr>
<th>Fields:</th>
<th>$L = (\nu, e)$</th>
<th>$L^c = (N^c, e^c)$</th>
<th>$\Phi = (H_u, H_d)$</th>
<th>$\Delta_L$</th>
<th>$\Delta_R$</th>
</tr>
</thead>
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<td>SU(2)$_L$</td>
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<td>2</td>
<td>3</td>
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<tr>
<td>SU(2)$_R$</td>
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<td>1</td>
<td>3</td>
</tr>
<tr>
<td>U(1)$_{B-L}$</td>
<td>-1</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>-2</td>
</tr>
</tbody>
</table>

Lepton Yukawas:  $\mathcal{L}_Y = y LL^c \Phi + \frac{f}{2} (LL\Delta_L + L^c L^c \Delta_R)$

(both $y$ and $f$ are 3x3 symmetric matrices)

VEVs:  - $v_R = <\Delta_R^0>$ breaks SU$_{221}$ into SU$_{21}$
  - $v = <\Phi^0>$ breaks SU$_{21}$ into U(1)$_{em}$
  - $v_L = <\Delta_L^0> \sim v^2/M_\Delta$ is induced by EW breaking

Mass matrix in ($\nu, N$) basis:  $M_\nu = \begin{pmatrix} v_L f & v y \\ v y & v_R f \end{pmatrix}$
**Left-Right symmetric seesaw**

\[ M_{\nu} = \begin{pmatrix} v_L f & vy \\ vy & v_R f \end{pmatrix} \]

**Seesaw mechanisms:**
- \( v \ll v_R \) (Type I)
- \( v_L \ll v \) (Type II)

Integrating out the super-heavy neutrinos \( N \):

\[ m_{\nu} = m_{\nu}^{II} + m_{\nu}^{I} = v_L f - v^2 y (v_R f)^{-1} y \]

**Type I and II seesaw contributions to light neutrino masses are strictly intertwined**

Several Left-Right models which are fully consistent up to GUT scale **do not contain** other sources of \( \nu \) mass
LR seesaw: the parameter space

\[ m_\nu = v_L f - v^2 y (v_R f)^{-1} y \]

- \( v^2 = (174 \text{ GeV})^2 \) (EWSB)
- \( 0 \leq v_L \leq \text{GeV} \) (\( \Delta \rho \approx - \frac{2 v_L^2}{v^2} \))
- \( \text{TeV} \leq v_R \leq M_{\text{Pl}} \) (no RH weak currents)
- \( 0 \leq (m_\nu)_{ij} \leq \text{eV} \): partially known from oscillations data
- \( 0 \leq y_{ij} \leq 1 \): in general unknown Yukawa couplings, but
  - Minimal SUSY LR: \( y = \tan \beta \ y_e \)
  - Minimal SO(10): \( y = y_u \)
- \( 0 \leq f_{ij} \leq 1 \): completely unknown Yukawa couplings

Bottom-up approach: what is the structure of the matrix \( f \)?
To what extent we can reconstruct \( M_R = v_R f \)?
Consider a matrix $f$ solution of the seesaw formula for a given set of all other parameters.

Define:

$$\hat{f} \equiv \frac{m_\nu}{v_L} - f$$

Then:

$$m_\nu = v_L \hat{f} - v^2 y (v_R \hat{f})^{-1} y$$

**Duality:** $f$ solution if and only if $\hat{f}$ is

Solutions of the seesaw equation come in pairs:

$$f = f_1, \hat{f}_1, f_2, \hat{f}_2, \ldots$$
• Seesaw formula **non-linear in** $f$: for 3 lepton generations, one finds **8 solutions for** $f$ (4 dual pairs)

→ The right-handed neutrino mass matrix has **8 possible structures**, $M_R = v_R f$

→ For a given $y$, **8 structures of** $f$ **induce the same** $m_\nu$

• One may derive a complete analytic resolution of the **non-linear polynomial system of equations for** $f_{ij}$

\[*m_\nu = v_L f - v^2 y (v_R f)^{-1} y*\]

method 1: Akhmedov & MF
A realistic numerical example

Tribimaximal mixing:
\[ \tan^2 \theta_{23} = 1 \]
\[ \tan^2 \theta_{12} = 0.5 \]
\[ \tan^2 \theta_{13} = 0 \]
No CP violation

Eigenvalues: -0.1, 0.2, 1
Normal hierarchy with \( \Delta m^2_{\text{sol}} / \Delta m^2_{\text{atm}} = 0.031 \)

Given all these inputs, the 4 dual pairs of f structures are determined

\[ m \equiv \frac{m_\nu}{v_L} = \begin{pmatrix} 0 & 0.1 & -0.1 \\ \ldots & 0.55 & 0.45 \\ \ldots & \ldots & 0.55 \end{pmatrix} \]

 Eigenvectors:

\[ v_L v_R = v^2 \] (natural when scalar potential couplings are of order 1)

neglect CKM-like rotations (both charged lepton and neutrino Yukawa couplings diagonal in the same basis)

\[ y_1 = 10^{-2} \quad y_2 = 10^{-1} \quad y_3 = 1 \] (inter-generation hierarchy analog to charged fermions)

\[ m_\nu = v_L f - v^2 y (v_R f)^{-1} y \]
The 8 reconstructed solutions

\[
\begin{align*}
\hat{f}_1 & \approx \begin{pmatrix}
0 & 0 & -0.1 \\
\cdots & 0 & 0 \\
\cdots & \cdots & 1.4 \\
0 & 0 & -0.1 \\
\end{pmatrix} \\
\hat{f}_2 & \approx \begin{pmatrix}
\cdots & 0 & 0 \\
\cdots & \cdots & 1.0 \\
0 & 0 & -0.2 \\
\end{pmatrix} \\
\hat{f}_3 & \approx \begin{pmatrix}
\cdots & -0.1 & 0.1 \\
\cdots & \cdots & -1.0 \\
0 & 0 & -0.2 \\
\end{pmatrix} \\
\hat{f}_4 & \approx \begin{pmatrix}
\cdots & 0 & -0.04 \\
\cdots & \cdots & -0.36 \\
\end{pmatrix}
\end{align*}
\]

\[
\begin{align*}
f_1 & \approx \begin{pmatrix}
0 & 0.1 & 0 \\
\cdots & 0.5 & 0.4 \\
\cdots & \cdots & -0.9 \\
0 & 0.1 & 0 \\
\end{pmatrix} \\
f_2 & \approx \begin{pmatrix}
\cdots & 0.6 & 0.3 \\
\cdots & \cdots & 1.5 \\
0 & 0.1 & 0.1 \\
\end{pmatrix} \\
f_3 & \approx \begin{pmatrix}
\cdots & 0.6 & 0.2 \\
\cdots & \cdots & -0.3 \\
0 & 0.1 & -0.1 \\
\end{pmatrix} \\
f_4 & \approx \begin{pmatrix}
\cdots & 0.5 & 0.4 \\
\cdots & \cdots & 0.9 \\
\end{pmatrix}
\end{align*}
\]
Features of the solutions

Consider a given pair of dual solutions:

\[ f_4 \approx \begin{pmatrix} -0.001 & 0.105 & -0.14 \\ \ldots & 0.56 & 0.49 \\ \ldots & \ldots & 0.88 \end{pmatrix}, \quad \hat{f}_4 \approx \begin{pmatrix} 0.001 & -0.005 & 0.04 \\ \ldots & -0.01 & -0.04 \\ \ldots & \ldots & -0.33 \end{pmatrix} \]

\textbf{Seesaw Duality:} \quad f_4 + \hat{f}_4 = m \approx \begin{pmatrix} 0 & 0.1 & -0.1 \\ \ldots & 0.55 & 0.45 \\ \ldots & \ldots & 0.55 \end{pmatrix}

- \textbf{f}_4 \textbf{ structure} has dominant 23-block; large (but non-maximal) 2-3 mixing
- \textbf{Dual structure} is hierarchical, with dominant 33-entry; small 2-3 mixing

\textbf{One seesaw type dominance} in \( m_{12}, m_{22}, m_{23} \):

- type II in the case of \( f_4 \), type I in the dual case.

\textbf{Mixed seesaw} in \( m_{11}, m_{13}, m_{33} \).
Right-handed neutrino masses

as a function of the absolute scale of light neutrino masses

$M_1 \ (GeV)$

$\nu_R$

$10^{15}$

$10^{14}$

$10^{13}$

$10^{12}$

$10^{11}$

$0.001$ $0.002$ $0.005$ $0.01$ $0.02$ $0.05$ $0.1$ $0.2$

$m_1 \ (eV)$

Solution $f_4$: solid lines
Solution dual to $f_4$: dashed lines

Mixed seesaw

Level-crossing

Normal hierarchy

Quasi-degeneracy

Dominant type II

Dominant type I
Right-handed neutrino masses
as a function of the Left-Right symmetry breaking scale $v_R$

$M_i (\text{GeV})$

- Dominant type I
- Dominant type II
- Mixed seesaw
- Level-crossing

Solution $f_4$: solid lines
Solution dual to $f_4$: dashed lines

$\left( \frac{10^{16} \text{GeV}}{v_R} \right)$
Baryogenesis via Leptogenesis

- Majorana mass term $M_R N N$ for super-heavy neutrinos $N_i$ violates Lepton Number.

- $N_1$ decays at $T \approx M_1$ out-of-equilibrium generating a lepton asymmetry by the interference between decay amplitudes at tree level and 1-loop: $\epsilon_L \sim [ \Gamma(N_1 \rightarrow LH) - \Gamma(N_1 \rightarrow L^*H^*) ]$

- Standard Model B+L violating effects at $T > v$ convert lepton into baryon asymmetry. Since $[n_B/s]_{\text{exp}} \approx 10^{-10} \leq 10^{-3} \epsilon_L$, one needs $\epsilon_L \geq 10^{-7} \div 10^{-6}$.
Leptogenesis in Left-Right models

• The needed lepton asymmetry may be produced either by the type I seesaw sector ($N_i$ decays), by the type II seesaw sector ($\Delta_L$ decays), or by their interplay.

• LR symmetry implies that the same matrix $f$ determines both $N_i$ masses and $\Delta_L$ coupling to leptons: more predictivity

• The 8 possible structures of $f$ can be discriminated by their ability to achieve Baryogenesis via Leptogenesis

• Seesaw duality provides new options to enhance the asymmetry: (i) solutions where $M_R$ is not hierarchical, (ii) solutions with quasi-degenerate masses, (iii) extra sources of asymmetry in the LR breaking sector, ...

Multiple options for leptogenesis


Solution \( f \)

Only CKM phase

Solution dual to \( f \)

Other CP-violating phases
Grand Unification à la SO(10)

\( SU(3)_c \times [SU(2)_L \times SU(2)_R \times U(1)_{B-L}] \subset SO(10) \)

All SM fermions + N’s sit in the same multiplet \( 16_F \)

Neutrino Majorana masses from a unique coupling:

\[
f \ 16_F 16_F 126_H \ni f (LL\Delta_L + L^c L^c \Delta_R) \]

Neutrino Dirac masses can receive several contributions:

\[
v y = \langle 10_H \rangle y_{10} + \langle 120_H \rangle y_{120} + \langle 126_H \rangle f
\]

Even if \( f \) contributes to \( y \) the seesaw can be written as:

\[
m'_L = v_L f - v^2 y' (v_R f)^{-1} y'
\]

Therefore there are always multiple solutions for \( f \).

Duality holds if only 10s and 126s (only 120s) contribute to \( y \).
Renormalizable Yukawas from one $10_H$ and one $126_H$ only

Babu, Mohapatra, Clark, Kuo, Nakagawa, Bajc, Senjanovic, Vissani, Melfo, Aulakh, Girdhar, Macesanu, Goh, Ng, Dutta, Mimura, Bertolini, Frigerio, Malinsky, ...

\[ \mathcal{L}_Y = 16_F \left( y \, 10_H + f \, 126_H \right) \overline{16}_F \]

However, $y$ and $f$ strongly constrained by charged fermion masses and CKM mixing angles

\[ M_u = y \langle 10_H \rangle^u + f \langle 126_H \rangle^u \]
\[ M_d = y \langle 10_H \rangle^d + f \langle 126_H \rangle^d \]
\[ M_e = y \langle 10_H \rangle^d - 3f \langle 126_H \rangle^d \]

The global fit of fermion masses and mixing (including neutrinos) is intricate and very constrained.

Most recent analysis: a perfect fit is possible, but the required heavy mass spectrum is incompatible with gauge coupling unification.

Aulakh & Garg, hep-ph/0512224
Bertolini, Malinsky & Schwetz, hep-ph/0605006
126_H plus two 10_H multiplets

1) **Up versus down**: two 10_H distinguish \( v_y = M_u \) from \( M_d = M_e \)
2) The 8 dual solutions for \( f \) may be derived **from neutrino sector**
3) For each viable \( f \), one may compute
   (i) lepton asymmetry
   (ii) lepton flavor violation bounds
   (iii) **correction to** \( M_d \neq M_e \), which remains difficult

126_H plus one 10_H and one 120_H

1) Fit with 10_H and 120_H alone \( M_u, M_d \) and \( M_e \)
   (there is some small tension for first generation masses).
2) Derive the 8 dual solutions for \( f \) **from neutrino sector**
3) Select the (possibly) unique structure for \( f \) which achieves a good **fit of** \( m_e, m_u, m_d \).
• The understanding of neutrino mass relies on the identification of its dominant source.

• If the new physics if Left-Right symmetric, type I+II seesaw stands up firmly as the unique candidate.

• Bottom-up reconstruction of the superheavy seesaw sector: duality among 8 different structures.

• Numerical & analytic reconstruction of the 8 structures allows to investigate different options for:
  - Baryogenesis via Leptogenesis
  - Grand Unified Theories
  - flavor symmetries, lepton flavor violation, etc...